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LETTER TO THE EDITOR

On the analytical soliton-like solution for the anisotropic Heisenberg model

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Abstract. It is shown that a recently presented analytical solution for the solitons in an anisotropic Heisenberg model does not satisfy the essential demands for being a soliton solution.

The problem of the exact analytical solution for solitons in the classical anisotropic Heisenberg ferromagnet was first presented by Tjon and Wright [1], who formulated the equation and studied its numerical solutions.

The Hamiltonian studied has the form:

$$H-H_0=-\mu f\sum_j (S_j^z-S)-J\sum_j (\boldsymbol{S}_j\cdot\boldsymbol{S}_{j+1}-S^2)-J\tau\sum_j (S_j^zS_{j+1}^z-S^2). \qquad (1)$$

Here $\boldsymbol{S}_i$ is the classical angular momentum vector of the length $|\boldsymbol{S}|=S$ associated with the site i of a periodic chain. μ is the magnetic moment, f the external magnetic field, J is the energy of the nearest-neighbour interaction and τ is the anisotropy parameter.

Škrinjar *et al* [2] have shown that the application of Holstein–Primakoff boson representation [3] combined with the application of boson coherent states [4] leads in the classical limit to the same equation which was put into the following form:

$$A_\xi^2[1+2\tau A^2(1-\tfrac{1}{2}A^2)] = \gamma_0 A^2[1+2\tau/\alpha^2\gamma_0 - V^2/4\gamma_0 - A/2(1+4\tau/\alpha^2\gamma_0)+(\tau/2\alpha^2\gamma_0)A^4. \qquad (2)$$

In obtaining this equation, it was assumed that the phase of the soliton has the form

$$\phi(x,t)=\Omega t+\varphi(\xi) \qquad \xi=x-vt \qquad (3a)$$

and the amplitude has the form

$$A(x,t)=A(\xi) \qquad (3b)$$

where v is the soliton velocity. Following notation used in [2]:

$$\gamma_0=\frac{S\Omega+\mu f}{J\alpha^2} \qquad V=Sv/(J\alpha^2)$$

where α is the lattice constant. It is important to notice that the relation to the standard interpretation in terms of spin components is established through the relation

$$A^2(\xi)=1-\cos\theta(\xi) \qquad (4)$$

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where θ is the polar angle measuring the deviation of the spin from the direction of z axis.

Quite recently, Lan and Wong (LW) [5] proposed that the solution for

$$B = A^2 \tag{5}$$

should be looked at in the form:

$$B = \frac{1}{a + b \cosh \mu\xi} \tag{6}$$

a, b and μ are the parameters to be determined from the differential equation (2) (expressed in terms of B). At this stage, it is important to notice that A is a real quantity, so necessarily $B > 0$, implying $a + b \geq 0$. (This corresponds to the maximal value of B at $\xi = 0$.) In fact, the case $a + b = 0$ leads to the result that diverges at the origin, so it cannot be accepted for the soliton solution. We are left with the condition

$$a + b > 0. \tag{7}$$

(In fact, from (4), (5) and (6), it follows that $a + b \geq 0.5$, but it is not relevant for the present discussion.)

The general idea is to substitute (6) into equation (2) and obtain the set of equations for a, b and μ. The following set is obtained [5]:

$$b^4(4\gamma_0 + 8\tau/\alpha^2 - V^2) = b^4\mu^2 \tag{8}$$

$$b^2\mu^2(2ab + 2\tau b) = (4\gamma_0 + 8\tau/\alpha^2 - V^2)4ab^3 - (2\gamma_0 + 8\tau/\alpha^2)b^3 \tag{9}$$

$$b^2\mu^2(a^2 - \tau + 2a\tau - b^2)$$
$$= (4\gamma_0 + 8\tau/\alpha^2 - V^2)6a^2b^2 + (2\tau/\alpha^2)b^2 - (2\gamma_0 + 8\tau/\alpha^2)3ab^2 \tag{10}$$

$$-b^2\mu^2(2\tau b + 2ab) = (4\gamma_0 + 8\tau/\alpha^2 - V^2)4a^3b + 4\tau ab/\alpha^2 - (2\gamma_0 + 8\tau/\alpha^2)3a^2b \tag{11}$$

$$-b^2\mu^2(a^2 - \tau + 2a\tau) = (4\gamma_0 + 8\tau/\alpha^2 - V^2)a^4 + 2\tau a^2/\alpha^2 - (2\gamma_0 + 8\tau/\alpha^2)a^3. \tag{12}$$

Here, a typographical error from [5] in equation (8) is corrected.

From this set, one can deduce the values of a, b and μ and also obtain additional constraints for the parameters of the system. Our aim here will be to show that the above system cannot lead to a solitonic solution.

From (8), one easily obtains

$$\mu^2 = 4\gamma_0 + 8\tau/\alpha^2 - V^2 \tag{13}$$

which is LW equation (54). From (9), one can obtain LW equation (55)

$$a = \tau + 1/\mu^2(\gamma_0 + 4\tau/\alpha^2) \tag{14}$$

or in a different form

$$2\gamma_0 + 8\tau/\alpha^2 = 2\mu^2(a - \tau). \tag{15}$$

Using (10), one can obtain LW equation (56):

$$b^2 = -5a^2 + 2\tau a + (\gamma_0 + 4\tau/\alpha^2)6a/\mu^2 - 2\tau/\alpha^2\mu^2 - \tau \tag{16}$$

but its combination with (15) leads to another useful expression

$$b^2 = a^2 - 4a\tau - \tau - 2\tau/\mu^2\alpha^2. \tag{17}$$

Using (15), equations (11) and (12) can be rewritten in the form:

$$-2b^2\mu^2(a+\tau)=4a^3\mu^2-6a^2\mu^2(a-\tau)+4\tau a/\alpha^2 \tag{18}$$

$$-b^2\mu^2(a^2+2\tau a-\tau)=a^4\mu^2-2a^3\mu^2(a-\tau)+2\tau a^2/\alpha^2. \tag{19}$$

(Notice that in the corresponding LW equation (58) $2\tau a^2/\alpha^2$ is written as 2τ.)

Combination of (17) and (18) now gives

$$2a+8a\tau+2\tau+4\tau/\alpha^2\mu^2=0 \tag{20}$$

while using (17) and (20) to rewrite (19), we obtain

$$4a+1+2/\alpha^2\mu^2=0. \tag{21}$$

This means that a must be negative.

Finally, if we use (21) to replace only the term linear in a in (17), we arrive at the conclusion that

$$a^2=b^2. \tag{22}$$

The first possibility, $b=a$ gives $a+b<0$, so it is not acceptable. The second one, $b=-a$ implies $a+b=0$ which also does not lead to a solitonic solution.

In this way, we have shown that the system of equations (8)-(12), leads to the solutions which either contradict the initial assumption ($B^2\geqslant 0$), or do not behave as soliton solutions, implying that the soliton solution cannot be found in the proposed form (6).

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References

[1] Tjon J and Wright J 1977 *Phys. Rev.* B **15** 3470
[2] Škrinjar M J, Kapor D V and Stojanović S D 1989 *J. Phys: Condens. Matter* **1** 725
[3] Holstein T and Primakoff H 1940 *Phys. Rev.* **58** 1098
[4] Glauber R J 1963 *Phys. Rev.* **131** 2766
[5] Lan H and Wong K 1990 *J. Phys. A: Math. Gen.* **23** 3923